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ScienceDirect

Procedia Engineering 199 (2017) 453–458

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

Experimental data based cable tension identification via nonlinear static inverse problem

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Abstract

This work proposes a new cable tension identification technique based on a static inverse method that, by coupling a universal cable model with displacement and strain sensors data, exploits the differences between the original cable equilibrium problem and that of the cable loaded by a suitable added mass. The formulated inverse problem thus defines a data misfit functional based on the differences in terms of transverse displacements and elongations between the two equilibrium configurations. The inverse problem is implemented in a two-step identification procedure. First, the axial stiffness and mass per unit length are kept constant and the length of the cable is approximately found via a simple line search algorithm using finite differences to estimate the functional derivatives. Second, the other physical parameters are assessed using an adjoint method for which the direct problem, the adjoint problem and the parameters sensitivities are found as derivatives of a Lagrangian functional with respect to dual variables, primary variables, and parameters, respectively. Due to the ill-conditioning nature of the problem, the proposed method does not allow an exact parameter identification but it does lead to an acceptable tension assessment. An experimental test campaign conducted on a multilayered stranded cable 21 m long and 22 mm in diameter subject to several tension levels confirms the relevance and operational feasibility of the proposed inverse method.

Peer-review under responsibility of the organizing committee of EURODYN 2017.

Keywords: cable, mixed formulation, inverse problem, tension identification

1 Introduction

Tension identification of cables has been widely studied over the years due to its key role in structural health monitoring. When no permanent monitoring system is installed on a bridge, three main methods exist for tension assessment: the use of jacks, the use of tensiometers and dynamical testing. Using jacks on a job site is costly and more likely to happen during a repair or an important maintenance operation (suspenders or anchorage replacements, suspension tuning operation...) for which an accurate tension assessment is not usually necessary. The tensiometer can be of great use for tension identification of small cables and ropes but is dependent on the cable type and has

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1877-7058
Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.013

to be carefully calibrated in the laboratory. Last but certainly not least, because of its easy application and low cost, dynamical testing for tension assessment has been extensively treated in the literature and used on sufficiently long cables. Despite the great differences between all the established methods in the way this relationship is found and used, they all rely on the combination of three parts: a cable model, some sensors and a post-treatment to obtain the natural frequencies of the cable. It is possible to embrace the variety of combinations looking at the following well-known references: [1], [2], [3], [4], [5]. A brief overview of dynamical testing for tension assessment is given in [6] where the reliability of the methods for cables longer than 19 m was confirmed. For shorter cables, physical uncertainties due to unknown boundary conditions or cable parameters (e.g., length or flexural stiffness) have set the tension assessment issue in terms of identification and inverse problems using the dynamics of tensed (and straight) beam as in [7], [8], [9].

To the authors knowledge, very few works considered the mass per unit length as an unknown of the problem. In practice, however, its value is likely to be known with an uncertainty around 5 to 10%. In the case of dynamical testing using the string theory, there is a direct consequence of this uncertainty on the tension identification precision.

In the present work, an inverse problem is formulated thanks to a data misfit functional based on the differences in terms of vertical displacements and axial elongations between two equilibrium states of the cable, namely, one loaded and the other free. It allows to find the tension in a cable without knowing precisely and *a priori* its physical parameters, namely its (stress-free) length L, its mass per unit length and its axial stiffness. The tension is expressed in terms of its corresponding safety factor $\gamma = Af_y/\breve{N}_{max}$, where f_y is the yield stress of the cable, A its cross-section area and $\breve{N}_{max}$ is the maximum value of the tension in the cable. All simulations were conducted in a Python environment embedded in the finite element plateform FEniCS [10].

2 Cable model: primal and mixed formulations of the direct problem

We briefly present the cable nonlinear mixed formulation used for tension assessment. More theoretical and implementation details can be found in [11]. The stress-free configuration $\mathbf{B}^0$ and the current configuration $\breve{\mathbf{B}}$ are described in the Newtonian basis $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$ with origin O. $\mathbf{B}^0$ is described by a curve $\mathbf{C}^0 = \{\boldsymbol{r}^0,\ s \in [0, L]\}$ parameterized by its arclength s whose length is L. Similarly, quantities $\breve{\mathbf{C}}$, $\breve{s}$, $\breve{\boldsymbol{r}}$ and $\breve{L}$ are defined for the current configuration $\breve{\mathbf{B}}$.

We define a space of smooth enough displacement fields H^1 over $[0, L]$ and the two spaces $\mathbb{V} = \{\breve{\boldsymbol{r}} \in H^1 |\ \breve{\boldsymbol{r}}(0) = \boldsymbol{o}$ and $\breve{\boldsymbol{r}}(L) = L_1\boldsymbol{e}_1 + L_2\boldsymbol{e}_2 + L_3\boldsymbol{e}_3\}$ and $\mathbb{V}_0 = \{\delta\breve{\boldsymbol{r}} \in H^1 |\ \delta\breve{\boldsymbol{r}}(0) = \boldsymbol{o}$ and $\delta\breve{\boldsymbol{r}}(L) = \boldsymbol{o}\}$. The problem is formulated as follows:

$$
\begin{aligned}
&\text{Find } \breve{\boldsymbol{r}} \in \mathbb{V} \text{ such as for every } \delta\breve{\boldsymbol{r}} \in \mathbb{V}_0\\
&\int_0^L (\breve{\boldsymbol{n}} \cdot \delta\breve{\boldsymbol{r}}_s)\ ds - \int_0^L \left(\breve{\boldsymbol{f}} \cdot \delta\breve{\boldsymbol{r}}\right) ds - [\breve{\boldsymbol{n}} \cdot \delta\breve{\boldsymbol{r}}]_0^L = 0,\\
&\breve{\boldsymbol{n}} = EA\breve{\Delta}\ \breve{\boldsymbol{r}}_s \text{ for } s \in [0, L].
\end{aligned}
$$

where the subscript s denotes differentiation with respect to s, the strain according to the Green-Lagrange measure is $\breve{\Delta} = \frac{1}{2}(\breve{\boldsymbol{r}}_s \cdot \breve{\boldsymbol{r}}_s - 1)$ and EA is the axial stiffness of the cable. This problem is a nonlinear minimization problem:

$$
\text{Find } \breve{\boldsymbol{r}} \in \mathbb{V} \text{ minimizing } J(\breve{\boldsymbol{r}}) = \int_0^L \left(\tfrac{1}{2}EA\breve{\Delta}^2 - \breve{\boldsymbol{r}} \cdot \breve{\boldsymbol{f}}\right) ds \tag{1}
$$

It can be completely solved thanks to the displacement only formulation given above. It is however interesting for data assimilation purposes to introduce the strain as an unknown imposing its definition as a constraint and by letting $\breve{T}$ be the associated Lagrange multiplier. Thus, the problem (1) reads:

$$
\begin{aligned}
&\text{Find } \breve{\boldsymbol{r}} \in \mathbb{V}, \breve{\Delta} \in \mathbb{P}_\Delta \text{ and } \breve{T} \in \mathbb{P}_T \text{ such as}\\
&J(\breve{\boldsymbol{r}}, \breve{\Delta}, \breve{T}) = \int_0^L \left[\tfrac{1}{2}EA\breve{\Delta}^2 - \breve{\boldsymbol{r}} \cdot \breve{\boldsymbol{f}} - \breve{T}\left(\breve{\Delta} - \breve{\Delta}^e\right)\right] ds \qquad (2)\\
&\text{is a saddle point and } \breve{\Delta}^e := \frac{1}{2}(\breve{\boldsymbol{r}}_s \cdot \breve{\boldsymbol{r}}_s - 1)
\end{aligned}
$$

By saddle point we mean that such $J(\breve{\boldsymbol{r}}, \breve{\Delta}, \breve{T})$ is a relative minimum for $\breve{\boldsymbol{r}}$ and $\breve{\Delta}$ and a relative maximum for $\breve{T}$. $\mathbb{P}_\Delta$ and $\mathbb{P}_T$ are suitable function spaces. The first variation $\delta J(\breve{\boldsymbol{r}}, \breve{\Delta}, \breve{T})$ of $J(\breve{\boldsymbol{r}}, \breve{\Delta}, \breve{T})$ shall be zero and gives:

$$\delta J(\breve{\boldsymbol{r}}, \breve{\Delta}, \breve{T}) = \int_0^L \left[\left(EA\breve{\Delta} - \breve{T}\right)\delta\breve{\Delta} + \breve{T}\breve{\boldsymbol{r}}_s \cdot \delta\breve{\boldsymbol{r}}_s - \breve{\boldsymbol{f}} \cdot \delta\breve{\boldsymbol{r}} - \left(\breve{\Delta} - \breve{\Delta}^e\right)\delta\breve{T}\right] ds = 0. \tag{3}$$

In (3), we recognize three sets of equations: (i) $EA\breve{\Delta} = \breve{T}$ is the constitutive equation that identifies the Lagrange multiplier associated with the constraint as the tension $\breve{T}$, (ii) $\breve{\Delta} = \breve{\Delta}^e$ is the strain-displacement relationship acting as a constrains here, and (iii) subsequent to integration by parts the strong form of the equilibrium problem $\left(\breve{T}\breve{\boldsymbol{r}}_s\right)_s + \breve{\boldsymbol{f}} = 0$ is recovered. It is clear that $J(\breve{\boldsymbol{r}}, \breve{\Delta}, \breve{T})$ is a Hu-Washizu functional [12]. Its second variation $\mathfrak{d}\delta J(\breve{\boldsymbol{r}}, \breve{\Delta}, \breve{T})$ is simply:

$$\mathfrak{d}\delta J(\breve{\boldsymbol{r}}, \breve{\Delta}, \breve{T}) = \int_0^L \left[\left(EA\mathfrak{d}\breve{\Delta} - \mathfrak{d}\breve{T}\right)\delta\breve{\Delta} + \mathfrak{d}\breve{T}\breve{\boldsymbol{r}}_s \cdot \delta\breve{\boldsymbol{r}}_s + \breve{T}\mathfrak{d}\breve{\boldsymbol{r}}_s \cdot \delta\breve{\boldsymbol{r}}_s - \left(\mathfrak{d}\breve{\Delta} - \breve{\boldsymbol{r}}_s \cdot \mathfrak{d}\breve{\boldsymbol{r}}_s\right)\delta\breve{T}\right] ds \tag{4}$$

which also gives the linearized version of the nonlinear problem about $\{\breve{\boldsymbol{r}}, \breve{\Delta}, \breve{T}\}$.

A suitable nondimensional form of the equations is obtained introducing the following nondimensional parameters:

$$L^* = L/l \qquad l^* = 1.0 \qquad \kappa = EA/EA_{th} \qquad m = \rho A l/EA_{th} \qquad \breve{T}^* = \breve{T}/EA_{th} \qquad \breve{\boldsymbol{r}}^* = \breve{\boldsymbol{r}}/l \qquad s^* = s/l$$

where $EA_{th} = 180000\ 10^6 \times 0.75 \times 0.25 \times \pi d_c^2$ N, d_c being the diameter of the considered cable and l its span. Henceforth we drop the stars for ease of notation. Note that mg is the nondimensional mass. The localized additional mass $\boldsymbol{f}_{mass}$ is also made nondimensional according to $\boldsymbol{f}_{mass}/EA_{th}$.

3 Inverse problem

3.1 Data misfit

The monitoring system is taken to consist of N strain gauges and M displacement sensors. Our goal is to recover the tension of the cable or the corresponding safety factor γ in its static configuration. To this end, we propose to use the response of the cable to an applied load (added weights) to study the following data misfit functional $\mathfrak{J}(L, \kappa, m) = J_{str} + J_{disp}$ between two configurations referred to by using superscripts a and b with:

$$J_{str}(L, \kappa, m) = \sum_{i=1}^{N} \frac{c_{str}}{2} \left[\left(\breve{\Delta}^{i,b} - \breve{\Delta}^{i,a}\right) - \left(\Delta^{i,b} - \Delta^{i,a}\right)\right]^2 \quad \text{and} \quad J_{disp}(L, \kappa, m) = \sum_{j=1}^{M} \frac{c_{disp}}{2} \left[\left(\breve{r}_1^{j,b} - \breve{r}_1^{j,a}\right) - \left(r_1^{j,b} - r_1^{j,a}\right)\right]^2,$$

where breve quantities are calculated via the direct problem (2) and other quantities are experimental measurements. Weighting factors c_{str} and c_{disp} are equal to 10^{12} and $10^4/d_c^2$, respectively, where d_c is the diameter of the cable. The numerical exploration of $\mathfrak{J}$ is presented in [11].

3.2 Finding the cable length

In accordance with the outcomes of the parametric studies presented in [11], we choose to address first the length search at fixed ρA and EA. In doing this, we will not be able to find the true set of parameters $L, \rho A, EA$, but the tension will be assessed correctly. A length search is carried out in a very simple way via a line search method. It uses the direct problem to numerically assess the derivative of $\mathfrak{J}$ with respect to L. Its pseudo code can be found in [11].

3.3 Finding the tension

Let L be the length of the cable determined by the line search method. To find the tension, a set of parameters (m, κ) that minimizes $\mathfrak{J}(m, \kappa)$ must be sought. This minimization problem is solved via the adjoint state method (a complete introduction on inverse problems can be found in [13]).

The **Lagrangian** $\mathfrak{L}$ is defined as follows:

$$\mathfrak{L} = \mathfrak{J} - J^b - J^a \tag{5}$$

where J^b is:

$$J^b = \int_0^L \left[\left(\kappa\breve{\Delta}^b - \breve{T}^b \right) \breve{\Delta}^{*b} + \breve{T}^b \breve{\boldsymbol{r}}_s^b \cdot \breve{\boldsymbol{r}}_s^{*b} - \left(\breve{\Delta}^b - \breve{\Delta}^{e,b} \right) \breve{T}^{*b} - \boldsymbol{f}^b \cdot \breve{\boldsymbol{r}}^{*b} \right] ds \tag{6}$$

and J^a has the same expression with superscripts a instead of b. We recall that $\breve{\Delta}^e := \frac{1}{2} (\breve{\boldsymbol{r}}_s \cdot \breve{\boldsymbol{r}}_s - 1)$ is the strain expressed in terms of displacements.

The **direct variational problem** is obtained via $\mathfrak{L}$ derivatives with respect to starred quantities (dual variables):

$$\delta J^b = \int_0^L \left[\left(\kappa\breve{\Delta}^b - \breve{T}^b \right) \delta\breve{\Delta}^{*b} + \breve{T}^b \breve{\boldsymbol{r}}_s^b \cdot \delta\breve{\boldsymbol{r}}_s^{*b} - \left(\breve{\Delta}^b - \breve{\Delta}^{e,b} \right) \delta\breve{T}^{*b} - \boldsymbol{f}^b \cdot \delta\breve{\boldsymbol{r}}^{*b} \right] ds = 0 \tag{7}$$

and δJ^a has the same expression with superscripts a instead of b.

The **adjoint problem** is obtained thanks to the derivatives of $\mathfrak{L}$ with respect to primal variables:

$$-\int_0^L \left[f^{\Delta,b} \delta\breve{\Delta}^b + \boldsymbol{f}^{r,b} \cdot \delta\breve{\boldsymbol{r}}^b \right] ds = \int_0^L \left[\left(\kappa\breve{\Delta}^{*b} - \breve{T}^{*b} \right) \delta\breve{\Delta}^b + \left(\breve{\boldsymbol{r}}_s^b \cdot \breve{\boldsymbol{r}}_s^{*b} - \breve{\Delta}^{*b} \right) \delta\breve{T}^b + \left(\breve{T}^b \breve{\boldsymbol{r}}_s^{*b} + \breve{T}^{*b} \breve{\boldsymbol{r}}_s^b \right) \cdot \delta\breve{\boldsymbol{r}}_s^b \right] ds \tag{8}$$

and

$$\int_0^L \left[f^{\Delta,a} \delta\breve{\Delta}^a + \boldsymbol{f}^{r,a} \cdot \delta\breve{\boldsymbol{r}}^a \right] ds = \int_0^L \left[\left(\kappa\breve{\Delta}^{*a} - \breve{T}^{*a} \right) \delta\breve{\Delta}^a + \left(\breve{\boldsymbol{r}}_s^a \cdot \breve{\boldsymbol{r}}_s^{*a} - \breve{\Delta}^{*a} \right) \delta\breve{T}^a + \left(\breve{T}^a \breve{\boldsymbol{r}}_s^{*a} + \breve{T}^{*a} \breve{\boldsymbol{r}}_s^a \right) \cdot \delta\breve{\boldsymbol{r}}_s^a \right] ds \tag{9}$$

where $f^{\Delta,b}$, $\boldsymbol{f}^{r,b}$, $f^{\Delta,a}$ and $\boldsymbol{f}^{r,a}$ can be found in [11].

The **sensitivities** are obtained thanks as the derivatives of $\mathfrak{L}$ with respect to m and κ:

$$\frac{\partial\mathfrak{L}}{\partial\kappa} = -\int_0^L \left(\breve{\Delta}^b \breve{\Delta}^{*b} + \breve{\Delta}^a \breve{\Delta}^{*a} \right) ds, \quad \frac{\partial\mathfrak{L}}{\partial m} = -\int_0^L g\boldsymbol{e}_1 \cdot \left(\breve{\boldsymbol{r}}^{*b} + \breve{\boldsymbol{r}}^{*a} \right) ds. \tag{10}$$

For implementation purposes, updates are performed via the Barzilai-Borwein method [14].

4 Experimental validation

An experimental validation was conducted using a steel multilayered stranded cable 21 m long and 22 mm in diameter. The test protocol considers the two static configurations a and b of the cable for 6 different tension levels in configuration a. The point load is applied via a 86.120 kg mass of steel as depicted in Fig. 1. In order to double check the tension obtained via the load cell, dynamical tests were performed for each tested tension level. The monitoring system consists of two accelerometers (DeltaTron type 4507 B 002 Brüel & Kjær ±70 m.s^{-2} peak), two displacement sensors (Intelligent-L300 Laser Sensor Keyence 160-450 mm), one load cell (FN2796 Measurement Specialties up to 400 kN), 4 strain gauges (Tokyo Sokki Kenkyujo Co. Sensors) as shown in Fig. 2. A drawing of the set-up can be found in Fig. 1.

The following protocol has been applied at 6 different tension levels from 55 kN to 205 kN: 1) Cable installation up to a specific tension (load cell), 2) static measurements, 3) mass positioning, 4) static measurements, and 5) mass removal.

In addition to the experimental validation of the proposed method, dynamical tests based on the string theory were performed for cross checking purposes. Fast Fourier Transforms were applied on the accelorometers data in order to obtain the cable vertical natural frequencies. In the string theory, the explicit relationship between the (constant) tension T in the cable and its first natural frequency f_1 is simply $T = 4f_1^2 l^2 \rho A$, where l is the length of the cable which is also its span. The use of the string theory is justified for validation purposes since it can be shown that in our testing conditions, its accuracy is almost constant throughout all tests and is driven by the precision of our knowledge

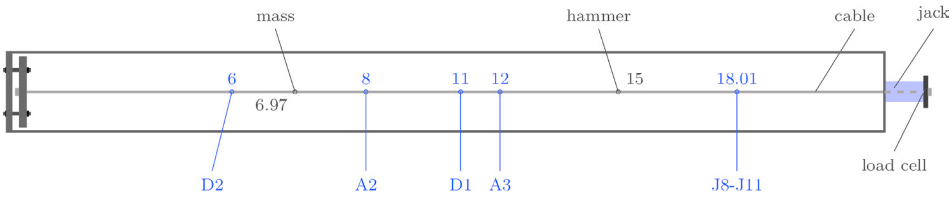

Fig. 1. Drawing of the test set-up. D = displacement sensor, A = accelerometer, J = strain gauges. Positioning is given in meters from the left anchorage

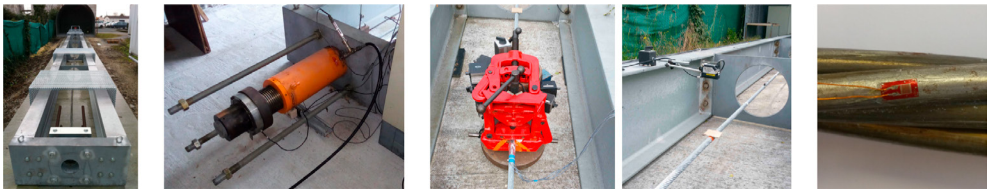

Fig. 2. Test bench. From left to right: bench structure, jack and load cell, additional mass to create a point load, displacement laser sensor, strain gauge.

of ρA. In fact, even for the lowest tested tension, we have: $\frac{dT}{T} = 2\frac{dl}{l} + 2\frac{df_1}{f_1} + \frac{d\rho A}{\rho A} \approx \frac{d\rho A}{\rho A}$. For the tests, we assume that the mass per unit length of cable is equal to $\rho A = 7810 \times 0.75\pi \times 0.011^2$ kg.m^{-1}, in accordance with the fill ratio commonly used for stranded cables.

Data from the tests can be found in [11]. Results based on data post-processing using both the string theory and our proposed inverse problem method are reported in Table 1. Paying attention to the load cell measurements $T_{load\ cell}$ and to the results obtained with the dynamical tests, we notice a large variation of the accuracy of the method from −10.9% to −6.0%. These results are in disagreement with the expected constant accuracy of the method, as previously discussed. According to the calibration tests, the accuracy of the load cell is close to 1 kN for high values of tensions and about 3 kN for low tensions. Trusting the highest load measure at 205 kN, it is possible to calibrate the dynamical method by correcting the mass per unit length of the cable to obtain a reference tension T_{ref} in the cable for all tests. We report in Table 1 the corresponding values.

At it can be seen, the correction leads now to a consistent and constant error for the dynamical tension assessment. Thanks to the load cell and the test at the highest level of tension, we can see that our estimation of the mass per unit length of cable was underestimated by −6%. More importantly, the proposed inverse problem achieves good accuracy in finding the cable tension for all 6 considered tension levels. When compared to the dynamical tests, the inverse method does not require a fine knowledge of the cable parameters and is therefore of great interest. In practice, it is even possible to assess the tension with the proposed inverse problem and then its mass per unit length thanks to the dynamical tests, thus improving its parameters identification.

Table 1. Tension assessment results for the 6 tested tensions.

	Units	55 kN	85 kN	115 kN	145 kN	175 kN	205 kN
$T_{load\ cell}$	[kN]	55.9	84.7	114.7	145.1	174.6	205.7
T_{ref}	[kN]	53.0	82.0	113.9	143.9	175.3	205.7
T_{inv}	[kN]	50.9	79.3	110.1	139.6	171.2	199.8
err_{inv}	[%]	−3.9	−3.3	−3.3	−2.9	−2.4	−2.9
T_{LPC35}	[kN]	49.8	77.1	107.1	135.2	164.8	193.3
err_{LPC35}	[%]	−6.0	−6.0	−6.0	−6.0	−6.0	−6.0

It might look expensive to advocate the use of added masses and displacement sensors that need be attached to a bridge deck and the cables. However, current maintenance procedures prescribe periodic man power cycles over the bridge often with heavy equipment (visual inspection). The tension evaluation process that we propose here does not need to be carried more than once. The outcome of the method might be viewed as the necessary preprocessing stage of the standard and very cheap dynamical testing methods as shown before.

5 Conclusions

In this work, we have proposed a new tension identification procedure via an inverse static problem using a non-linear mixed formulation for elastic cables statics. To the authors' best knowledge, tension identification in cables is rarely addressed via an inverse static approach. Moreover, only few works consider the length, the stiffness and the mass per unit length of cable as unknowns of the problem despite their importance and the fact that they are only roughly known in practice. From a modeling point of view, the use of a mixed formulation for a geometrically exact cable is also unusual in the literature regardless of the advantages one could harness from the direct calculation of strains and stresses viewed as primary variables [11]. The inverse problem for tension identification is obtained thanks to a data misfit functional $\mathfrak{J}(L, \rho A, EA)$ relying on the differences in terms of displacements and strains measured at a few control points on the cable between two equilibrium configurations. Despite the ill-posed nature of the inverse problem caused by the multiplicity of its solutions, it has been shown in [11] that the correct tension could be found following a two-step procedure. First, setting the axial stiffness and the mass per unit length to their initial guess values, the length L of the cable is approximately found via a simple line search algorithm using finite differences to estimate the functional derivative of $\mathfrak{J}(L)$ with respect to L. Second, the two other physical parameters are assessed using an adjoint method for which the direct problem, the adjoint problem and the parameters sensitivities are found as derivatives of a Lagrangian functional with respect to dual variables, primary variables, and parameters, respectively.

An experimental validation was carried out using a multilayered stranded cable 21 m long and 22 mm in diameter. 6 different tension levels ranging from 55 kN up to 205 kN were considered. A comparison with a traditional dynamical testing method showed the reliability of the proposed method and its operational interest; tensions were correctly assessed within less than 4% error without requiring a fine knowledge of the cable's stiffness and mass per unit length.

Acknowledgements

The work of Walter Lacarbonara was partially supported by a PRIN MIUR Grant (Grant No. 2010BFXRHS-002).

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